

On possible memory effects in tests of Bell inequalities

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Abstract

It is shown that memory effects in experiments measuring correlations in entangled photon pairs are not able to produce a relevant loophole for the test of local hidden variables theories.

1 Introduction

The Bell inequalities[1] are necessary conditions for the existence of local realistic (or local hidden variables) models able to interpret the results of some specific experiments. In particular the experiments involving coincidence measurements made by two separated parties, Alice and Bob, on entangled pairs of particles. The relevance of Bell's work is that there are idealized experiments where quantum mechanics predicts violations of the inequalities, which strongly suggests that for some actual experiments no local hidden variables model is possible. The contradiction between local realism and quantum mechanics is known as "Bell's theorem". The question whether the contradiction also exists for actual, rather than ideal, experiments has led people to perform many Bell tests. Until recently all tests have provided results compatible with both quantum mechanics and local realistic models, the latter compatibility deriving from the existence of loopholes in the tests. Last year several experiments have been performed that, for the first time, blocked simultaneously the two most relevant loopholes, detection inefficiency and locality[2], [3], [4]. The purpose of this note is to see whether a loophole still remains, namely the *memory loophole*.

The memory loophole derives from the fact that tests of the Bell inequalities require performing many similar experiments on different entangled pairs of particles in order to estimate the probabilities from the frequencies actually measured. In practice there is a series of trials with a given experimental set-up, the trials being separated from each other by short time intervals. It is usually assumed that any hidden variables associated with the n th particle pair would be independent of measurement choices and outcomes for the first $(n-1)$ pairs. Models which violate this assumption exploit the possible memory effects. The strongest type of violation uses a 2-sided memory loophole, in which the hidden variables for pair n can depend on the previous measurement choices and outcomes in both wings of the experiment.

The memory loophole has been studied by Barrett et al.[5]. The authors concluded that, although in principle the memory loophole implies a slight flaw in existing analyses of Bell experiments, the data still may refute local realistic models. However Barrett et al. proved the irrelevance of the memory loophole for tests of the CHSH (Bell type) inequality[6], whilst the recent photon experiments test the CH-Eberhard inequality[7]. The point is that the CHSH inequality (2) is appropriate for event-ready detectors, like in the experiment by Hensen et al.[2], where the probabilities may be calculated on a well defined set of entangled particle pairs. In contrast the CH-Eberhard inequality (1) is used when the set of pairs is not well defined, as in the recent photon experiments. In these experiments the photon pairs are produced via parametric down conversion in a nonlinear crystal. The consequence is that when neither Alice nor Bob detect a photon in a trial they cannot know if either photons arrived to the parties but none was detected or no photon was produced in the source during the trial. In fact the second case is by far more probable. Thus in the photon experiments we cannot determine true probabilities but only relative probabilities. The question arises whether the memory loophole is also irrelevant in these conditions. In the following I give an affirmative answer to the question.

In photon experiments in more detail the Bell test is divided into a series of trials. During each trial Alice and Bob randomly choose between one of two measurement settings, denoted a and a' for Alice and b and b' for Bob, and record either a “+” if they observe any detection event or a “0” otherwise. The details of that type of Bell test may be seen, for instance, in the experiments by Shalm et al.[3] or by Giustina et al.[4]. Alice may get two possible results in each one of the two possible measurements of her photon, and similar for Bob. Therefore there are 16 coincidence probabilities that

might be determined. In practice the Bell test requires just 4 that may be combined in the CH/Eberhard inequality[7], namely

$$0 \leq B \equiv P(0+ | ab') + P(+0 | a'b) + P(++ | a'b') - P(++ | ab). \quad (1)$$

The terms $P(++ | ab)$ and $P(++ | a'b')$ correspond to the probability that both Alice and Bob record detection events $(++)$ when they choose the measurement settings ab or $a'b'$, respectively. Similarly, the terms $P(+0 | ab')$ and $P(0+ | a'b)$ are the probabilities that only Alice or Bob record an event for settings ab' and $a'b$, respectively.

For comparison I write the CHSH inequality, that is

$$E(ab) + E(a'b) + E(ab') - E(a'b') \leq 2, \quad (2)$$

where $E(ab)$ denotes the expectation value of the product of the outcomes of the measurements with the settings a and b . With the notation of eq.(1) we have

$$E(ab) = P(++ | ab) + P(00 | ab) - P(+0 | ab) - P(0+ | ab).$$

Actually the inequalities (1) and (2) are equivalent, in the sense that one of them is fulfilled if and only if the other one holds true, provided that the label “0” has the same meaning in both. But this is not obviously the case when we compare event ready experiments, e. g. by Hensen et al.[2] with experiments with photons produced via parametric down conversion, like those of Giustina et al.[4] or Shalm et al.[3]. Indeed in the latter type of experiment the CHSH inequality is useless because most of the times no photon arrives at the detectors, whence we would have

$$E(ab) \simeq P(00 | ab) \simeq 1.$$

As a result the left side of eq.(2) is very close to 2 and, what is more relevant, it cannot be calculated precisely because the fraction of trials corresponding to no photons emitted from the source is uncertain.

In spite of these difficulties it is possible to prove that the test of local hidden variables is reliable. The proof follows. The joint probability $P(x, y; a, b)$ that the particles yield the outcomes x and y when subjected to the measurement with the settings a and b respectively is given, according to Bell[1], by

$$P(x, y; a, b) = \int d\lambda \rho(\lambda) P(x; a, \lambda) P(y; b, \lambda), \quad (3)$$

where, in our notation, x, y might be either $+$ or 0 . Eq.(3) may be taken as the definition of local hidden variables model. Now let us assume that there are memory effects such that the hidden variables, λ , and the action of the measuring devices in the trial n depend on all previous trials, $j = 1, 2, \dots, n-1$. This means that we shall substitute the following

$$P_n(x, y; a, b) = \int d\lambda_n \rho_n(\lambda_n) P_n(x; a, \lambda_n) P_n(y; b, \lambda_n), \quad (4)$$

for eq.(3). That is, all possible memory effect may just change the nature and the probability distribution of the hidden variables, λ , and the probability of outcome in the measurement for given settings, a and b . If the probabilities of the 4 outcomes $ab, ab', a'b, a'b'$ are $1/4$, as insured by the random choice, then inserting eq.(??) in eq.(1) we get the CH-Eberhard inequality

$$0 \leq B_n \equiv P_n(0+ | ab') + P_n(+0 | a'b) + P_n(++ | a'b') - P_n(++ | ab). \quad (5)$$

This happens if an entangled pair of photons is produced in the source in the n th trial, but if no photons are produced then eq.(1) gives

$$B_n = 0.$$

In any case for a set of trials we shall have

$$\sum_j B_j \geq 0,$$

where $\{j\}$ represents the trials chosen for the test of the CH-Eberhard inequality. In practice it is common to choose, within some time interval, all trials such that at least one photon is detected (either by Alice or by Bob). The absence of bias in the random choice of the settings is essential for the proof. For instance the local model

$$P(+; a, \lambda) = P(+; b, \lambda) = P(+; a', \lambda) = 1, P(+; b', \lambda) = 0,$$

gives $B = 0$, taking eqs.(3) and (1) into account, for unbiased random choices. However if the choices a and b have probabilities $(1+\varepsilon)/2$ each and the choices a' and b' probabilities $(1-\varepsilon)/2$, then we get $B = -\varepsilon$ in apparent violation of eq.(1).

We conclude that for photon tests involving the CH-Eberhard inequality the possible memory loophole is irrelevant provided that the test involves a large enough number of trials. For a small number there may be fluctuations that could give a wrong answer. The study of fluctuations will not be made here, it would be similar to the study made by Barrett et al.[5] in relation with the CHSH inequality.

References

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